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Reading to Learn Mathematics

Inverse Functions and Relations

Pre-Activity How are inverse functions related to measurement conversions?

Read the introduction to Lesson 7-8 at the top of page 390 in your textbook.

A function multiplies a number by 3 and then adds 5 to the result. What does the inverse function do, and in what order?

Reading the Lesson

1. Complete each statement.

- If two relations are inverses, the domain of one relation is the _____ of the other.
- Suppose that $g(x)$ is a relation and that the point $(4, -2)$ is on its graph. Then a point on the graph of $g^{-1}(x)$ is _____.
- The _____ test can be used on the graph of a function to determine whether the function has an inverse function.
- If you are given the graph of a function, you can find the graph of its inverse by reflecting the original graph over the line with equation _____.
- If f and g are inverse functions, then $(f \circ g)(x) = \underline{\hspace{2cm}}$ and $(g \circ f)(x) = \underline{\hspace{2cm}}$.
- A function has an inverse that is also a function only if the given function is _____.
- Suppose that $h(x)$ is a function whose inverse is also a function. If $h(5) = 12$, then $h^{-1}(12) = \underline{\hspace{2cm}}$.

2. Assume that $f(x)$ is a one-to-one function defined by an algebraic equation. Write the four steps you would follow in order to find the equation for $f^{-1}(x)$.

- _____
- _____
- _____
- _____

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Study Guide and Intervention

Inverse Functions and Relations

Find Inverses

| | |
|--------------------------------------|---|
| Inverse Relations | Two relations are inverse relations if and only if whenever one relation contains the element (a, b) , the other relation contains the element (b, a) . |
| Property of Inverse Functions | Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$. |

Example

Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y .

$$x = \frac{2}{5}y - \frac{1}{5}$$

Inverse

$$5x = 2y - 1$$

Multiply each side by 5.

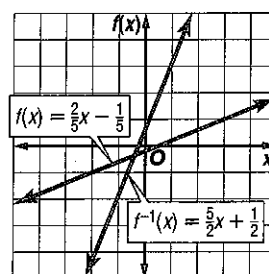
$$5x + 1 = 2y$$

Add 1 to each side.

$$\frac{1}{2}(5x + 1) = y$$

Divide each side by 2.

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

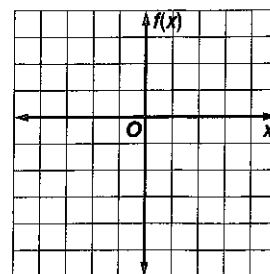
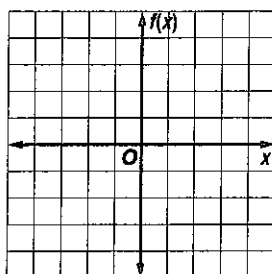
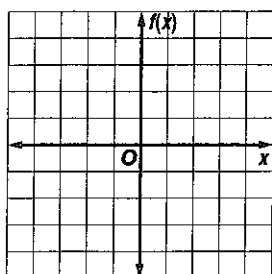
**Exercises**

Find the inverse of each function. Then graph the function and its inverse.

1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$



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Skills Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(3, 1), (4, -3), (8, -3)\}$

2. $\{(-7, 1), (0, 5), (5, -1)\}$

3. $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

4. $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5. $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

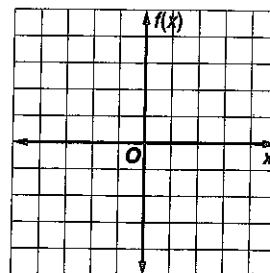
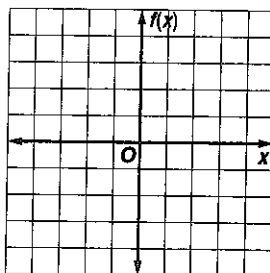
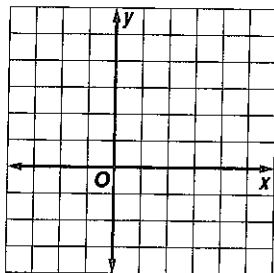
6. $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

SHOW WORK!
Find the inverse of each function. Then graph the function and its inverse.

7. $y = 4$

8. $f(x) = 3x$

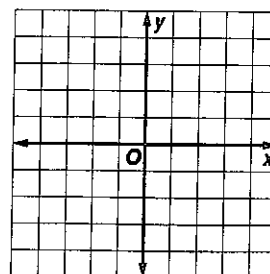
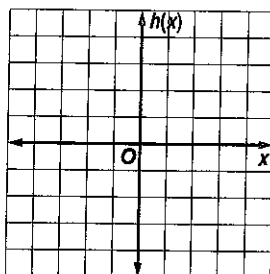
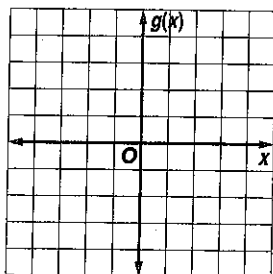
9. $f(x) = x + 2$



10. $g(x) = 2x - 1$

11. $h(x) = \frac{1}{4}x$

12. $y = \frac{2}{3}x + 2$



Determine whether each pair of functions are inverse functions. (using composition)

13. $f(x) = x - 1$

$g(x) = 1 - x$

14. $f(x) = 2x + 3$

$g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$

$g(x) = \frac{1}{5}x + 1$

16. $f(x) = 2x$

$g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$

$g(x) = \frac{1}{6}x + 3$

18. $f(x) = 8x - 10$

$g(x) = \frac{1}{8}x + \frac{5}{4}$

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Practice**Inverse Functions and Relations**

Find the inverse of each function.

1) $h(x) = \sqrt[3]{x} - 3$

2) $g(x) = \frac{1}{x} - 2$

3) $h(x) = 2x^3 + 3$

4) $g(x) = -4x + 1$

5) $g(x) = \frac{7x + 18}{2}$

6) $f(x) = x + 3$

7) $f(x) = -x + 3$

8) $f(x) = 4x$

- 9) **MEASUREMENT** The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

REMODELING For Exercises 17 and 18, use the following information.

The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs \$17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

- 10) Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys?

- 11) What will the new flooring cost the Clearys?